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$$\theta_1^3\theta_2+\theta_1\theta_2^3+\theta_1^3\theta_3+\theta_1\theta_3^3+\theta_2^3\theta_3+\theta_2\theta_3^3=(I/2a^2)(\theta_1\theta_2+\theta_1\theta_3+\theta_2\theta_3)=-I^2/8a^4.$$

$$\theta_1^3\theta_2^2+\theta_1^2\theta_2^3+\theta_1^3\theta_3^2+\theta_1^2\theta_3^3+\theta_2^3\theta_3^2+\theta_2^2\theta_3^3=-(\theta_1^2\theta_2^2\theta_3+\theta_1^2\theta_2\theta_3^2+\theta_1\theta_2^2\theta_3^2)$$

$$=-\theta_1\theta_2\theta_3(\theta_1\theta_2+\theta_1\theta_3+\theta_2\theta_3)=-IJ/16a^5.$$

$$\therefore A+B+C=\tfrac{1}{4}(80HI-507aJ).$$

$$AB+AC+BC=\frac{1}{64a^2}(342732a^4J^2-28561a^4I^3+179456a^2H^2I^2-262144H^4I$$
$$-27040a^3HIJ-245760aH^3J).$$

$$ABC=\frac{1}{64a^3}(12943360a^3H^3J^2-4826809a^6J^3+33554432H^6J-22970368a^2H^4IJ$$
$$-1142440a^5HIJ^2+3655808a^4H^2I^2J).$$

$$\therefore\frac{a^{12}\Pi}{4096}=-[(2125764a^6J^2+524880a^5HIJ^2-3326427a^6J^2+\tfrac{1}{16}(27761292a^6J^2$$
$$-2313441a^6J^3+14535936a^4H^2I^2-21233664a^2H^4I-2190240a^5HIJ$$
$$-19906560a^3H^3J)+\tfrac{1}{16}(12943360a^3H^3J-4826809a^6J^2+33554432H^6$$
$$-22970368a^2H^4I-1142440a^5HIJ+3655808a^4H^2I^2).$$

$$\therefore a^{12}\Pi=256(2313441a^6I^3-3723875a^6J^2-5065400a^5HIJ+6963200a^3H^3J$$
$$+44204032a^2H^4I-18191744a^4H^2I^2-33554432H^6).$$

GEOMETRY.

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306. Proposed by J. SCHEFFER, A. M., Kee Mar College, Hagerstown, Md.

Find the length of the perpendicular let fall from the point in space (5, 6, 7) upon the line $x=2z-3$, and $y=-3z+1$.

Solution by G. B. M. ZERR, A. M., Ph. D., Parsons, W. Va.

Let $\left.\begin{matrix}x=sz+p\\y=tz+q\end{matrix}\right\}$ be the line in space, and m, n, l the point in space. $m, n, l=A$, the vertex of a triangle; $p, q, 0=B$, the point where line pierces the xy plane; $\dfrac{pt-qs}{t}$, 0, $-\dfrac{q}{t}=C$, the point where line pierces the xz plane.

$$c=AB=\surd[(m-p)^2+(n-q)^2+l^2].$$

$$b=AC=\sqrt{\left(\frac{mt-pt+qs}{t}\right)^2+n^2+\left(\frac{lt+q}{t}\right)^2}.$$

$$a=BC=(q/t)\surd(1+s^2+t^2).$$

The perpendicular from the vertex A on the side BC is

$$p=\sqrt{c^2-\left(\frac{a^2+c^2-b^2}{2a}\right)^2}.$$

$$\therefore p=\sqrt{(m-p)^2+(n-q)^2+l^2-\left(\frac{(m-p)s+(n-q)t+l}{\surd(1+s^2+t^2)}\right)^2}$$

$$=\sqrt{\frac{(mt-pt-ns+qs)^2+(sl-m+p)^2+(tl-n+q)^2}{1+s^2+t^2}}.$$

In the problem, $m=5$, $n=6$, $l=7$, $s=2$, $t=-3$, $p=-3$, $q=1$.

$$\therefore p=\sqrt{\frac{(34)^2+(6)^2+(26)^2}{14}}=\tfrac{1}{7}\sqrt{(6538)}=11.55.$$

Also solved by Mary R. Beck, A. H. Holmes, and J. Scheffer, and the Proposer.

307. Proposed by WALTER D. LAMBERT, 416 B Street N. E., Washington, D. C.

A family of planes containing a common line intersects a sphere. Find the orthogonal trajectories of the traces. An analytic solution is preferred.

Solution by F. H. SAFFORD, Ph. D., The University of Pennsylvania.

Let the sphere and family of planes be respectively,

$$x^2+y^2+z^2-2az=0,\quad z+c=\lambda x\ldots(1),$$

in which λ is the parameter of the planes, so that the common line is

$$z+c=x=0\ldots(2).$$

The following equations define an inversion in space,

$$x=\frac{2a^2x'}{x'^2+y'^2+z'^2},\quad y=\frac{2a^2y'}{x'^2+y'^2+z'^2},\quad z=\frac{2a^2z'}{x'^2+y'^2+z'^2}\ldots(3).$$

The result of (3) applied to (1) is, after reduction,

$$z'-a=0,\quad c(x'^2+y'^2)+ca^2+2a^3-2a^2\lambda x'=0\ldots(4).$$

Thus the circles defined by (1) have become the circles in (4), all lying in one plane. The family of circles orthogonal to (4) is

$$z'-a=0,\quad c(x'^2+y'^2)-ca^2-2a^3-2a^2\mu y'=0\ldots(5),$$

in which μ is the parameter.

Solving (3) for x', y', z', gives formulas for inverting (4) and (5). The former, of course, becomes (1) again, while the latter becomes

$$x^2+y^2+z^2-2az=0,\quad z(a+c)-ac+\mu ay=0\ldots(6).$$

It is a property of inversion that angles remain invariant, so that the system (1) and (6) is orthogonal and (6) is the desired trajectory, composed of circles of which the respective planes have the common line

$$z(a+c)-ac=y=0\ldots(7).$$